Maxwell’s Equations from Continuous Differential to Finite Differential Form

Three Dimensional Maxwell’s Equations

The pertinent Maxwell’s Equations for a version of the 1D FDTD form first principles begins with:

In three dimensions these equations boil down to:

Two Dimensional Maxwell’s Equations:

Taking these equations into two dimensions. Assuming that there is no variation in the z-direction so all partial derivatives of fields with respect to z equal zero and the structure extending to infinity in the z-direction with no change in the shape or position of its transverse cross-section:

The transverse magnetic or TM mode is for

The transverse electric mode is for

These equations can be leapfrogged together to solve for a particular scenario.

One Dimensional Maxwell’s Equations:

Taking these modes into one dimension,

TM assuming no variation in the y-direction we have:

TE assuming no variation in the y-direction we have:

Taking the initial conditions of we can obtain the equations:

These are the big two equations that we’ve been working up to.

These solutions can be found using assumptions for the other two dimensions with similar logic and should be relatively easy to derive with the solutions shown above.

From Continuous to Finite Differentials:

Translating these two equations from a continuous differential into a finite differential can come in multiple forms depending on how accurate you want to be.

Taking a Taylor series of a function around a point we obtain:

For a function u around a point solving for the previous and next point a distance from the point we would obtain:

And

Performing the feat of magic that is linear algebra, one can solve for any derivative they desire to any order of accuracy. The most common are the first and second derivatives to second order accuracy as:

And

Using these finite difference formulas, Maxwell’s equations in 1D for the TE mode become (ignoring :

So these are the general equations that one would use to solve for an EM wave propagating in the x-direction. One could provide a source function at some point and observe the propagation of the wave from that source. One interesting thing to note is that the variables mu, epsilon, rho and sigma are all variable in space and must be found at each index i.

The relationship between and is also special and should try as hard as possible to reach the magic relationship where where c is the speed of light in that particular medium. Typically one of these variables should be solved for and one of them determined by the user. I like to pick the spatial component and solve for the temporal component but in theory it is arbitrary.